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RESISTANCE OF THE ELECTRODES OF A RAILGUN WITH A DISTRIBUTED JUMPER

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Results of a calculation of the resistance of railgun electrodes with distributed and concentrated current jumpers are reported. The problem was solved using the quasistationary equation of magnetic field diffusion. Results of exact and approximate solutions are given. For the complicated pattern of current distribution on the boundary, it is proposed that the voltage drop rather than the electrode resistance be calculated.

The resistance to the current flow through railgun electrodes with a current jumper moving along the electrodes differs from the resistance in the "stationary" design. This is due to the fact that the current is not distributed over the entire cross section of the electrode but is limited by a narrow area adjacent to the working surface. The effect of a "fast" skin layer and its manifestations have been considered in a number of papers (see, for example, [1, 2]). The resistance, in particular, was calculated in [3]. A diagram of the plane problem is shown in Fig. 1: a current passes through the electrodes within the skin layer δ and through the jumper having point contact I with the electrodes in the longitudinal-section plane. Solution was performed using the quasistationary equation of magnetic-field diffusion in a coordinate system attached to the jumper. In the approximation of a "boundary layer," the variability of the transverse component of the current density j_y in the x direction is far less than the variability of the longitudinal component j_x in the y direction. This equation can be written as

$$\mu_0\sigma V\frac{\partial B}{\partial x}=\frac{\partial^2 B}{\partial y^2}, \tag{1}$$

where V is the velocity of motion of the electrode's mass relative to the contact, μ_0 is the magnetic permeability of vacuum, σ is the conductivity of the medium, and B is the magnetic-field induction.

Assuming that the magnetic-field induction depends only on one self-similar variable $\eta \approx y/\sqrt{x}$, it is possible to reduce Eq. (1) to an ordinary differential equation of the second order. Solving this equation, we obtain the following expression for the longitudinal current density:

$$j_x=\frac{J}{b}\sqrt{\frac{\mu_0\sigma V}{\pi x}}\exp\Big(-\frac{\mu_0\sigma V y^2}{4x}\Big) \tag{2}$$

(b is the width of the electrode and J is current flowing through the electrode). Hence it follows that the characteristic cross-sectional dimension of the current-flow region in the electrode is $\delta = J/bj_x(y=0) = \sqrt{\pi x/\mu_0\sigma V}$.

In deriving relation (2), we used the condition that the cross-sectional dimension of the electrode is much greater in height than δ.

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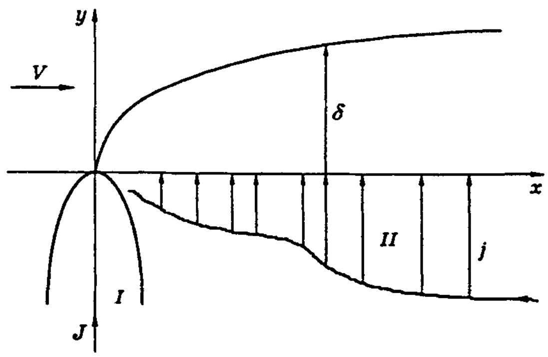

Fig. 1. Current flow pattern between the electrodes and the jumper: I and II are concentrated and distributed jumpers.

The resistance R to the current passage through the electrode is determined from the expression for Joule heat release in the cross section of the electrode,

$$Q = b\int_0^\infty j_x^2 \frac{dy}{\sigma}, \tag{3}$$

written in standard electrotechnical form as

$$Q = j_x^2\left(\frac{dR}{dx}\right). \tag{4}$$

Hence, $R = 1/b\sqrt{2\mu_0 V x/\pi\sigma}$.

A similar approach is applicable for the case of a distributed jumper II (see Fig. 1). We assume that the dependence $B(x, y)$ has the form

$$B = f(x)\beta(\eta), \tag{5}$$

where $\eta = y\mu_0\sigma V/2x$ is a self-similar variable.

Substitution of relation (5) into the initial equation (1) shows that a solution in the form (5) can be obtained only if

$$f(x)_- = ax^\alpha, \tag{6}$$

where a and α are constants. In this case, the equation for β has the form

$$\ddot{\beta} + \eta\dot{\beta} - 2\alpha\beta = 0. \tag{7}$$

The particular case $\alpha = 0$ corresponds to the solution described above [see formula (2)]. The general solution of Eq. (7) can be represented in elementary functions if 2α is an integer ($2\alpha = n$) [4]:

$$\beta = e^{-(\eta^2/2)}\frac{d^n}{d\eta^n}\left[e^{\eta^2/2}\left(c_1 + c_2\int e^{\eta^2/2}d\eta\right)\right]$$

(c_1 are c_2 are integration constants).

Assuming, as before, that the cross-sectional dimension of the electrode is much larger than the characteristic cross-sectional dimension of the zone of magnetic-field concentration, we obtain the boundary condition

$$\beta \to 0 \quad \text{for} \quad \eta \to \infty.$$

Hence it follows that $c_1 = 0$, and c_2 can be combined with the constant a. Thus, the expression for β takes

TABLE 1

n	γ_{exact}	γ_{approx}	ε, %
0	1	0.8862	11.4
1	1.3013	1.2533	3.69
2	1.5621	1.5350	1.73
3	1.7902	1.7724	1.00
4	1.9941	1.9817	0.62

the form

$$\beta = e^{-(\eta^2/2)} \frac{d^n}{d\eta^n}\left[e^{\eta^2/2} \int_\eta^\infty e^{-(\eta^2/2)} d\eta\right].$$

From the Maxwell equations

$$j_x = \frac{1}{\mu_0}\frac{\partial B}{\partial y}, \qquad j_y = -\frac{1}{\mu_0}\frac{\partial B}{\partial x} \tag{8}$$

we obtain

$$j_x = -\sqrt{\frac{\sigma V}{2\mu_0 x}}\, f(x)\frac{d\beta}{d\eta}, \qquad j_y = -\frac{f(x)}{\mu_0 x}\left(\beta - \frac{\eta}{2}\frac{d\beta}{d\eta}\right). \tag{9}$$

Using (3) and (4) and taking into account that on the surface of the electrode ($y = 0$)

$$B = B_0 = -\frac{\mu_0 J}{b} \tag{10}$$

(J is the local current strength in the electrode cross section), we obtain

$$R = \frac{1}{b}\sqrt{\frac{2\mu_0 V x}{\pi\sigma}}\,\gamma(n), \tag{11}$$

where the factor $\gamma(n)$ depends on the law of current passage through the boundary between the electrode and the jumper:

$$\gamma(n) = \frac{\sqrt{\pi}}{\beta^2(0)}\int_0^\infty \left(\frac{d\beta}{d\eta}\right)^2 d\eta.$$

Table 1 gives results of exact and approximate calculations of γ for several values of n and the relative error of the approximate calculation of ε. It can be seen that for the case of a distributed jumper ($n \neq 0$), the resistance of the electrode is higher than for the case of a concentrated jumper ($n = 0$).

The exact solution is applicable for a limited class of laws of current passage through the boundary when the normal current density is the same exponential function of the length throughout the working surface of the electrode.

In real situations, there can be several jumpers and the current flow through the zones of contact with the jumpers can obey different laws.

We consider an approximated solution of the problem using a method similar to the Goodman's method of "heat-balance integral" [5], which has been successfully used in solutions of complicated problems of heat transfer (see, for example, [6, 7]). We integrate Eq. (1) over the y coordinate, using the above-mentioned condition that the cross-sectional dimension of the electrode is much larger than δ:

$$\mu_0\sigma V \frac{d}{dx}\int_0^\infty B\, dy = -\left.\frac{\partial B}{\partial y}\right|_{y=0}. \tag{12}$$

Let the magnetic-induction distribution be specified as

$$B = B_0 \exp\left(-\frac{y}{\delta}\right), \tag{13}$$

where B_0 is a known quantity defined by relation (10) and δ is an unknown function of the x coordinate.

Substituting (13) into (12), we obtain

$$\frac{d}{dx} B_0 \delta = \frac{B_0}{\mu_0 \sigma V \delta}. \tag{14}$$

In this case, according to (8), we have $j_x = -(B_0/\mu_0\delta)\exp(-y/\delta)$, and, hence, using (10), from (3) and (4) we obtain

$$\frac{dR}{dx} = \frac{1}{2b\sigma\delta}. \tag{15}$$

The dependence $\delta(x)$ is found from (14):

$$\frac{d}{dx}(J\delta)^2 = \frac{2J^2}{\mu_0 \sigma V}. \tag{16}$$

The error of this calculation can be determined by comparing it with the exact model. Within the framework of the exact solution according to (6) and (9), we have $j_y(0) \approx x^{\alpha-1}$ and $j_y \approx x^{\alpha}$. Then, relation (16) leads to

$$\delta^2 = \frac{2}{n+1}\frac{x}{\mu_0 \sigma V},$$

and from (15) we obtain

$$R = \frac{1}{b}\sqrt{\frac{n+1}{2}\frac{\mu_0 V x}{\sigma}}.$$

Using (11), we find that in the approximated model, $\gamma(n) = \sqrt{(\pi/4)(n+1)}$. As n increases, the difference between the approximate and accurate calculations decreases. In particular, when the current transfer is uniform ($n = 2$), the difference is about 2%.

For a complicated pattern of the current flow distribution on the boundary, it is convenient to find the voltage drop rather than the resistance of one or another segment of the electrode. For this, relation (4) should be written as $Q = -J(dU/dx)$, where U is the potential in the volume of the electrode. Then, using (3), (10), and (13) we obtain $dU/dx = B_0/(2\mu_0\sigma\delta)$. Using (14), we find that

$$U = U_0 + \frac{V}{2} B_0 \delta \tag{17}$$

(U_0 is the potential for $\delta = 0$, i.e., on the leading edge of the first jumper).

The dependence of $B_0\delta$ on the x coordinate is determined from (14):

$$(B_0\delta)^2 = \frac{2\mu_0}{\sigma V b^2}\int_0^x j_x^2\, dx. \tag{18}$$

Combining (17) and (18), we obtain the required solution for the voltage drop on a segment of the electrode.

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